tion, only, are approximately correct for the actual case of location wherein both phase and intensity are altered.

The work thus far clearly shows that with frequencies 100 to 1200 d.v phase is the *chief* factor in localization with pure tones. For the lower frequencies the intensity effect is practically nil, and at all frequencies within the range considered the effect must be small. These conclusions are quite contrary to the view commonly held and appearing in psychological literature and texts. But they are worthy of confidence because based on *quantitative* results, and not on personal judgment. There remains a great amount of experimental work covering an extension of the frequencies and the employment of complex tones. Strictly speaking the localization herein mentioned applies only to 90° either side of the median plane and in front. The removal of this limitation will probably prove relatively simple after the experiments are completed. The above report while covering a limited range, nevertheless does make a definite advance in our understanding of the relative importance of the factors, phase and intensity, in binaural localization, and furnishes a basis upon which the future experiments may depend.

THE MEASUREMENT OF SMALL TIME INTERVALS AND SOME APPLICATIONS, PRINCIPALLY BALLISTIC[1]

By L. T. E. Thompson, C. N. Hickman and N. Riffolt

Ballistic Institute, Clark University

Communicated by A. G. Webster, February 25, 1920

Precision in time calibration is of importance in many classes of research. Several types of apparatus have been developed for this purpose. Some of these, such as the Helmholtz pendulum, permit extremely accurate reproduction of a stated interval but less exact absolute determination. A method proposed and extensively used by A. G. Webster has given satisfactory, absolute determinations of intervals as small as a millionth of a second.[2] The recording apparatus is electrical and the calibration is accomplished by a careful measurement of the height of fall of a projectile.

The Le Boulengé and the Aberdeen chronographs are in general use, the former having a falling projectile and the latter a rotating drum. With any apparatus involving a rotating drum, it is necessary to maintain the speed of the motor driving it within certain limits and to secure a dependable calibration, usually placed on the record as it is being made. Various instruments have been devised to accomplish this purpose.[3] A tuning fork producing a photographic or mechanically inscribed record of a sinuous form is perhaps the commonest example.

Means for applying discontinuously a record of a fork or other vibrator

have been used. For example, a very successful method was recently developed at the Bureau of Standards by Duncan and Curtis in which straight lines were produced across the photographic film by the interruption of light passing through slits attached to the tuning fork.

In the ballistic investigations which have been conducted at Clark University during the past two years, a great many of the records obtained have required such a time scale; for example the pressure-time curves for a gun, the curves for the vibration of the gun barrel, and later the velocity measurements which are described in this paper. The Bureau method was first adopted. We have been able, however, to develop a new apparatus which permits the placing of equally sharp lines across the film at very high frequencies. This is of considerable importance in the small arms ballistic work, inasmuch as it is necessary to have the film moving at high speed.

The apparatus for the time records is shown in figures 1 and 2. The tuning fork has a small needle attached to one prong (a weight being attached to the other) against which there bears a light stiff filament of steel ribbon held in place by a massive frame attached to the base mounting

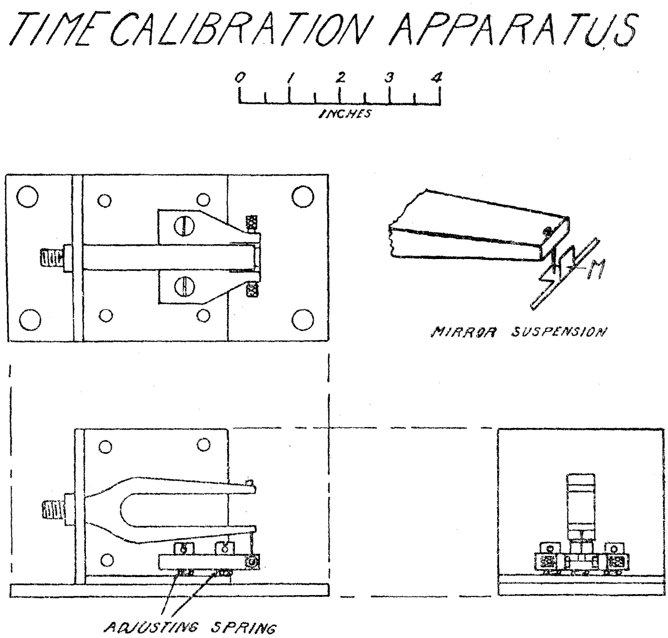
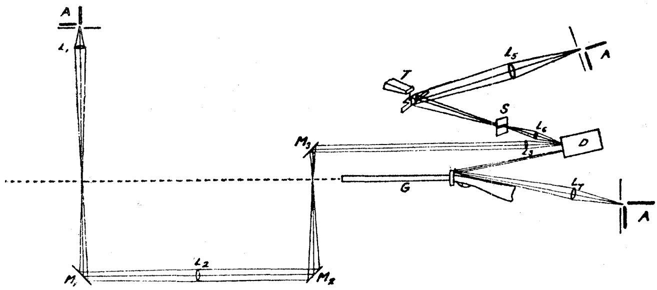


FIG. 1

of the tuning fork. (The filament is practically a duplicate of the one used in Professor Webster's Phonometer to record the motion of the vibrating disk.)[4] Upon this filament is mounted a small mirror. Light is reflected from it and focused on a stationary slit. The fork is struck with an elec-

trical hammer and allowed to vibrate freely, and in so doing causes the image of the slit in front of the arc to move up and down across the stationary slit. The stationary slit, then, is an intermittent source of light, the image of which is focused at a convenient place on the moving film. The optical system should be so designed that the beam only slightly more than covers the mirror. A new fork has been constructed giving about two thousand lines per second. There is no reason why this should not be as high as ten to

BALLISTIC APPARATUS



M PLANE MIRROR S SLIT
L LENS D ROTATING DRUM
P PRESSURE GAUGE G GUN
T TIME CALIBRATION APPARATUS

FIG. 2

twelve thousand lines per second. The velocity at the neutral position (where the slit is placed) is a maximum and includes the frequency as a factor. As it is increased the amplitude may decrease and still permit the same exposure of light through the slit as at the lower frequencies. Mechanically, the essential condition to be fulfilled in the design is that the natural frequency of the filament upon which the mirror is mounted should be high in comparison with that of the fork.

Most of the methods which are at present used in ballistics for the measurement of velocity involve the passage of the projectile through screens of one sort or another, breaking or making electrical circuits. The method which we have devised does not require that any material be touched by the projectile in its flight, but simply the passage twice through a beam of light. It is found that it is only necessary to observe the flight of the projectile over a space of from 3 to 5 feet to get a precision which is certainly greater than that needed for ordinary purposes.

The beam of light obtained from a slit is controlled by the optical system (fig. 2) in such a way that it is focused twice, each time in the path of the projectile, being reflected by the mirrors M_1, M_2 and M_3 and finally brought to the photographic surface on the rotating drum. The image is a short sharp line. It is desirable to have for both this and the calibrating system a short focus lens. As the film rotates, the image of the slit will

trace a narrow ribbon on the film surface. As the projectile passes through the ribbon in the two places successively an interruption of light occurs and a dark section appears in the ribbon at each of the corresponding instants. This is in general a longitudinally distorted image of the projectile and if the speed of the film were adjusted to a certain value it would be a miniature reproduction of its cross-section. Some records which have been obtained are shown in figure 3. The measurements have been

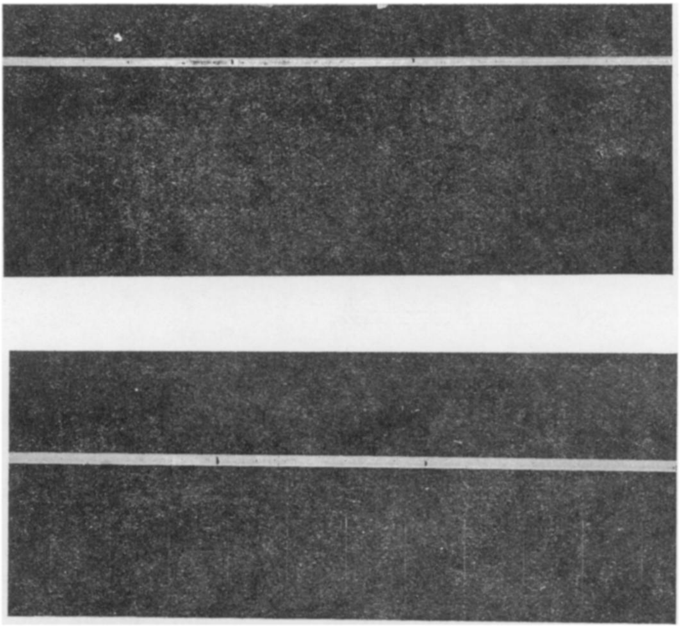


FIG. 3

made from the base line of the projectile in each case and the sharpness of this permits a determination of the length between these lines with a precision which is fully equal to that of the time calibration. The only observations, then, that are required are those of length. Each of these can be carried out to one part in ten thousand if necessary. The fact that a permanent record is provided and that no electrical apparatus is used also adds to the dependability of the results. A compact portable form has been designed for field use.

A general apparatus for ballistic investigation is shown in figure 2. This includes a pressure gauge[5] as well as the velocity equipment. With one discharge of the gun a fairly complete ballistic record is obtained on the film mounted on the rotating drum D. The velocity at the muzzle, the pressure-time curve in the barrel, including the pressure when the projectile leaves, that at maximum value, the time calibration, and, by

analysis of these curves, the velocity-time and pressure-space curves, as well as data regarding the resistance of the barrel, are all available.

An extensive application of this procedure is to be made in a determination of the resistance of air at various densities, to the flight through it of bodies moving with relatively high velocities. Heretofore, measurements have been made out-of-doors, shooting over a comparatively long range in order to get the desired accuracy. By making use of this velocity apparatus we are able to make two determinations in a length which is to be obtained in a building of laboratory size. The resistance is measured by the work which is done in the passage of the projectile from one point to another through the medium. This will require the two determinations of the velocity for the change in kinetic energy. It is necessary that the velocity be found with a precision which is sufficient to give the difference in the squares of the velocities with an acceptably small error. For a Government thirty caliber ($^1/_{50}$ lb.) projectile the resistance of the air at normal pressure is such that the velocity will decrease an amount of the order of 225 feet per second in travelling 300 feet from the muzzle. At densities greater than normal the decrease will, of course, be greater and at pressures less than normal (which will be obtained as described below) the decrease will be less than this.

$$\frac{\int_{s_1}^{s_2} R ds}{s_1 - s_2} = \bar{R} = \frac{m}{2_s}(v^2 - v_o{}^2)$$

is the mean value of the retardation over the distance of observation (100–200 feet).

An approximate idea of the effect upon the final value of the resistance of an error in the measurement of the velocity of one part in ten thousand is obtained by an examination of the above expression.

$$\frac{\delta R}{R} = \frac{2v\delta v}{v^2 - v_o{}^2} + \frac{2v_o\delta v_o}{v^2 - v_o{}^2} + \ldots = \frac{4 \times 2500 \times {}^1/_4}{650000} + \ldots \text{approximately, or about } 0.4\%.$$

Here $\Delta v = 125$; $v_o = 2650$; $v = 2525$; $\delta v = \delta v_o = {}^1/_4$. At higher pressures the error will be still less and at pressures down to one-half normal the substitution in this expression shows that the error for a single observation will still be below 1%.

Inasmuch as the ratio of the resistance of the medium to the weight of the projectile decreases as the diameter increases and in direct proportion, the precision which would be expected with large projectiles would be correspondingly less. Nevertheless, by increasing the distance of observation the same order of accuracy would be possible with projectiles up to perhaps 3 inches in diameter. If the firing is done in a tube of from 1 to 3 hundred feet in length, having a diameter sufficient to make observation possible without great inconvenience, the air pressure in this tube being controlled by a pump, the observations of the

velocities ranging perhaps from 1200 feet per second up to 3500 or 4000 feet per second can be made at each end of the tube. As long as the velocity of the projectile is greater than that of sound, no interference with the measurements will result from a disturbance reflected from the walls of the tube.

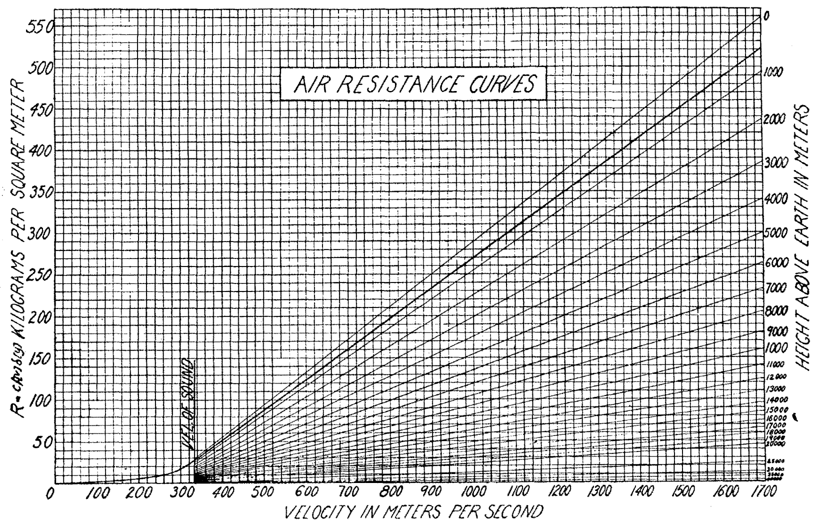
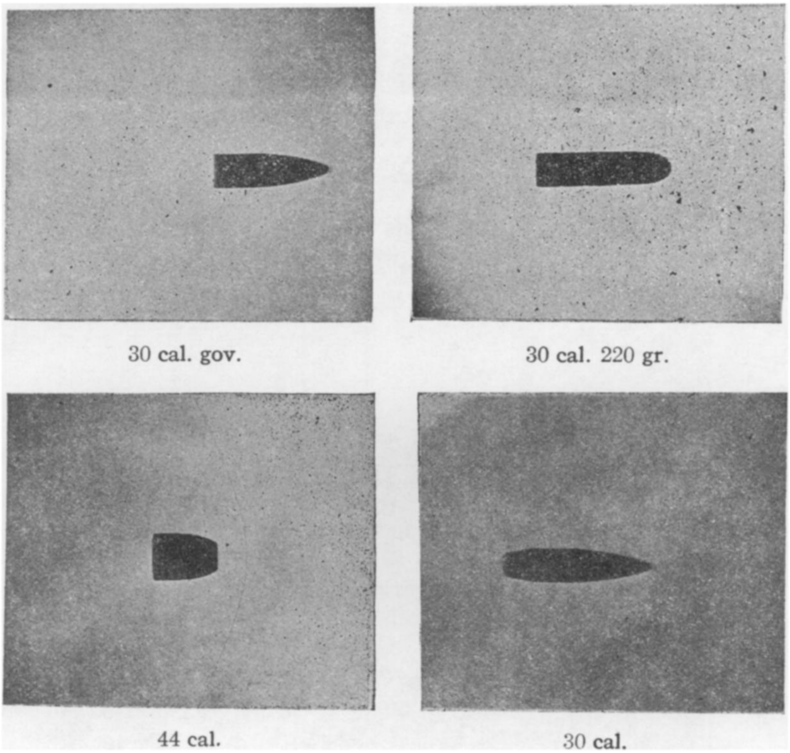


FIG. 4

Figure 4 is a graphical representation of the resistance as a function of the velocity as determined from the measurements made by at least five European observers, their data being averaged by the Italian ballistician, General Siacci. The retardation appears to be a linear function beyond the velocity of sound. Multiplying the datum line of these curves by different factors depending upon the density of the air gives the other curves for various heights. These were used in trajectory computations on long range guns of the type brought out by the Germans to shell Paris.[6] The experiments outlined will be adequate to make a precise determination of the values for the resistance for small projectiles over a considerable part of the usable portions of this graph for the modern rifles of high velocity. The fact that this problem is not the exact equivalent hydrodynamically of the one for very large projectiles does not materially lessen the value or interest of the results of such an investigation with small projectiles.

An important problem is the accurate determination of the effect of varying the form of the projectile. The question of ultimate or optimum form for a given use should be experimentally determinate. The records show not only the velocity but the manner in which the projectile is travelling. Also, it is likely that the precision and the complete control of the conditions of experiment possible would lead to satisfactory results

as to the variation of the so-called form factor with the velocity. If the results indicate an entirely new law of resistance for each form of projectile, as has been suggested, this fact should be evident. In this case there will not be a linear relation between the datum line for a projectile of one form over its range of velocities and that for another projectile with a different form. The fact that a projectile in actual flight may gradually change its cross-sectional surface presented in the direction of flight owing to precession (and thus produce a gradual change in the "law") is an argument for making the measurements in a short distance.



30 cal. gov. 30 cal. 220 gr.

44 cal. 30 cal.

FIG. 5

Another ballistic application of the method of determining velocities is in connection with penetration. The energy loss, and even the law of retardation, that is to say the resistance of a solid with the passage of a body through it under given conditions, can be found through spark photography and successive determinations of the velocity. Here it is practically necessary to be able to measure the velocity in a few feet, this apparatus thus being particularly adapted.

While the ballistic application of the experiments is perhaps most important, yet the general investigation of the hydrodynamical aspects is of considerable interest. It is not necessary to restrict the bodies

which are hurled through a medium to those of practical ballistic design or dimensions. Coupled with the spark method of obtaining instantaneous photographs of the projectile and the waves connected with its flight, it is possible to examine the condition of the medium and the effect of change in body shape for a great range of conditions.

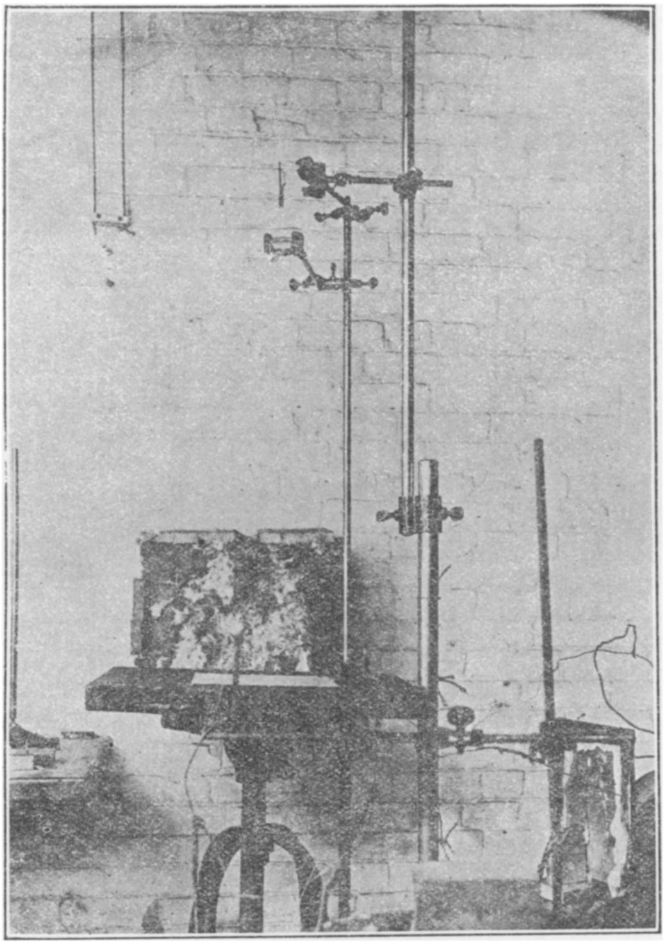


FIG. 6

The energy given up by a body in motion is dissipated in at least three ways—through skin friction, gas compression,[7] and production of the turbulent region particularly evident in the rear (see spark photographs, fig. 5). In order to formulate a theory of retardation it is necessary to have quantitative information regarding these factors and their relative importance. This information is very difficult to obtain.

Another application of the method of time calibration and of determining velocities comes in an entirely different field, namely, the measurement of the velocity of light. With a range of about 40 or 50 miles, which is available, a simple apparatus of this type will provide a record comparing the instant of arrival of light directly from the source (one of short duration, such as a spark or very rapidly moving slits) and that which has gone out and back over the 50 miles. The record will be similar to those which have been shown and obtained in much the same way. A precision which is at least as great as that obtained by the other methods which have been used for the measurement of the velocity of light is to be expected and in addition the permanent record, which will be given by a deflection on the film of the order of two centimeters, this representing the time taken for the light to travel the 50 miles.

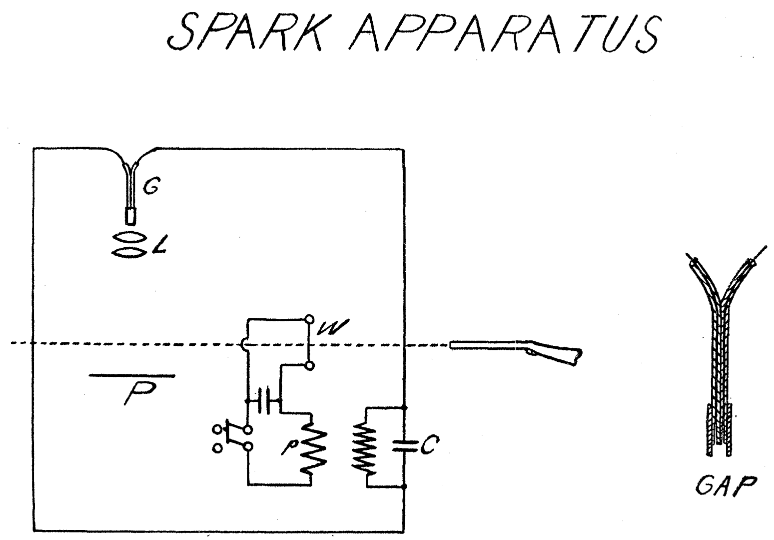


FIG. 7

The shadowgraphs (fig. 5) of projectiles in flight were made by use of the apparatus shown in figure 6, a diagram of which is shown in figure 7. The spark is obtained by breaking a small wire (w) through which the current passes to the primary of a large coil. The current is applied just before firing the gun, and in this way a very small wire can be used, not having time to fuse. The secondary condenser C controls the time interval elapsing between breaking the wire and the passing of the spark through the gap G which is constructed as shown in the figure. This makes a good "point" source and one always in the same place. The lens L is quartz. The position of the projectile on the plate can be controlled to perhaps 1 centimeter by moving the wire (w) with respect to the plate. This corresponds to a time interval of $^1/_{75,000}$ second.

[1] Contribution from the Ballistic Institute, Clark University, No. 7.

[2] Webster, A. G., "On A Means of Producing a Constant Angular Velocity," *Amer. Jour. Sci.*, **3,** May, 1897 (383–386).

[3] Webster, A. G., "An Experimental Determination of the Period of Electrical Oscillations," *Physic. Rev.*, **6,** 1898 (297–314); Hubbard, J. C., "An Experimental Determination of the Period of Electrical Oscillations," *Physic. Rev.*, **2,** 1, 1913 (247–249).

[4] Webster, A. G., *Proc. Nat. Acad. Sci.*, **5,** May 1919 (163–166).

[5] Webster, A. G. Thompson, L. T. E. "A New Instrument for Measuring Pressures in a Gun," *Proc. Nat. Acad. Sci.*, **5,** p. 258-263.

[6] Webster, A. G., "Some Considerations on the Ballistics of a Gun of Seventy-five-Mile Range," *Proc. Amer. Phil. Soc.*, **58,** p. 373–381. (Note by A. G. W. It should be stated that the linear law is not now considered exact. M. Lugot, of the Commission de Gâvre, informed the writer that the curves should rise a little above the straight lines, but the error is small.)

[7] Experiments to measure the compression are now in progress.—A. G. W.

CHARCOAL ACTIVATION

By H. H. Sheldon

Ryerson Physical Laboratory, University of Chicago

Communicated by R. A. Millikan, February 20, 1920

The results obtained by Dr. Harvey B. Lemon on the variations due to heat treatment in the adsorption of gases by charcoal,[1] although published but recently, were obtained some time ago. There seemed two possible explanations of these results—either that the structure of the

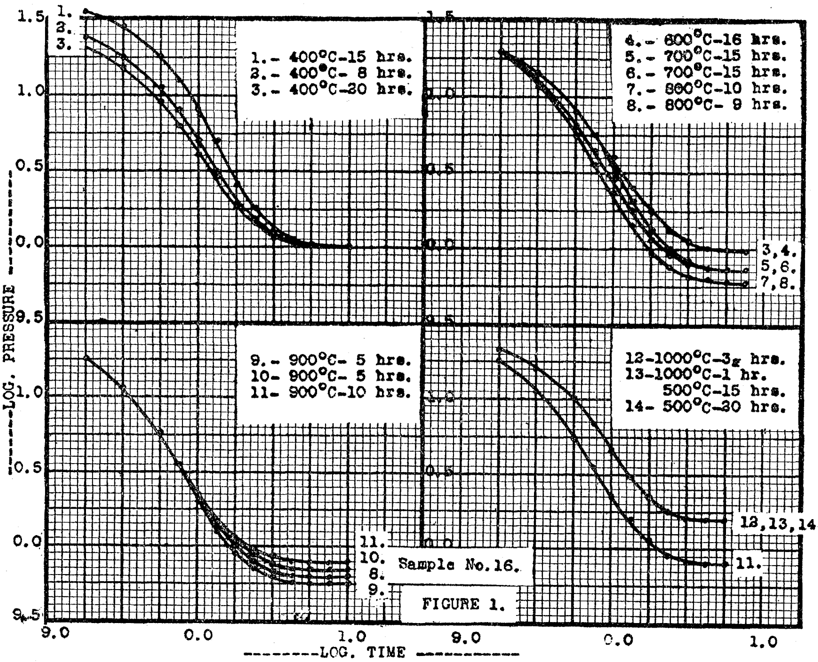


FIGURE 1.